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# A THREE-VALUED MODAL TENSE LOGIC FOR THE MASTER ARGUMENT

SEIKI AKAMA, TETSUYA MURAI AND SADAAKI MIYAMOTO

*Abstract*

The Master Argument was shown by Diodorus Cronos to conclude that nothing is possible that neither is true nor will be true and that therefore every (present) possibility must be realized at a present or future time. It leads to logical determinism. Prior tried to reconstruct the argument by means of modal tense logic. As a consequence, Prior proposed several branching time tense logics to resolve the fallacy of the Master Argument. In this paper, we propose a three-valued modal tense logic with a Kripke semantics to defend Prior's original argument.

## 1. *Introduction*

*The Master Argument* was shown by Diodorus Cronos to conclude that nothing is possible that neither is true nor will be true and that therefore every (present) possibility must be realized at a present or future time. It leads to *logical determinism* (or *fatalism*), which says that what is necessary at any time must be necessary at all earlier times. The logical reconstruction of the Master Argument was done by Prior [7] by means of *modal tense logic* which is a logical system with tense and modal operators. An alternative reconstruction was also found in Rescher [8]. As a consequence, Prior proposed several branching time tense logics to resolve the fallacy of the Master Argument.

Our starting point in this paper is that the difficulty with the Master Argument lies not in particular axioms but in a (standard) semantics for modal tense logic. Therefore, we employ a three-valued semantics without moving to non-standard modal tense logic based on branching times. This possibility was already worked out by Prior, but the attempt did not seem successful. By using the semantics based on Kleene's weak three-valued logic, the Master Argument cannot hold, in which future contingents occur.

The rest of this paper is as follows. In section 2, we give a quick review of the Master Argument and Prior's logical reconstruction. In section 3, we propose a three-valued modal tense logic $Q_t^m$ with a Kripke semantics and reveal that the Master Argument is not justified in $Q_t^m$. In section 4, we compare our approach to others in the literature. The final section gives some conclusions and defends our approach.

## 2. *What is the Master Argument?*

The Master Argument, as usually understood, was supported by the Megarean philosopher Diodorus Cronos, and its logical analysis bothered philosophers for many years. The gist of the argument is consists in the claim that the following three propositions cannot all be true (cf. Prior [7, p. 32ff]):

(M1) Every true proposition concerning the past is necessary.
(M2) The impossible does not follow from the possibility.
(M3) Something that neither is nor will be is possible.

Before giving a formal argument, we must address at least two points. One issue is: what is a "proposition" in the argument? The other is: what does it mean by "follows" in the argument? For the first question, the proposition is understood as a logician usually assumes, i.e. the sentence that is either true or false. This is the interpretation in classical logic, but it is not the case if one uses non-classical logics, in particular, many-valued logics. For the time being, however, we use the term "proposition" in classical sense.

As to the second question, there are three options. The first is temporal interpretation in which "follows" means "follows after". This interpretation is the underlying basis of the reconstruction of the argument in Rescher [8]. The second is material implication in classical logic. The third is some kind of entailment relation. Here, we adopt the third option. Therefore, the statement that $A$ *follows* $B$ is identified with the statement that $B$ *always materially implies* $A$. On these grounds, it is useful to employ modal tense logic for the representation of the Master Argument. Such an analysis was in fact done by Prior, who symbolized the argument by means of modal and tense operators. The argument is expressed in modal tense logic as follows.

(D1) $\mathrm{P}A \to \mathrm{LP}A$
(D2) $\mathrm{L}(A \to B) \to (\neg \mathrm{M}B \to \neg \mathrm{M}A)$
(D3) $(\neg A \wedge \neg \mathrm{F}A) \to \neg \mathrm{M}A$

Here (D3) is the negation of (M3). Thus, (D3) reads "what neither is nor will be true is not possible". We here use standard notation rather than Prior's Polish notation. And each operator used here has an obvious meaning.

Prior additionally used the following two extra assumptions:

(D4) $\mathrm{L}(A \rightarrow \mathrm{HF}A)$
(D5) $(\neg A \wedge \neg \mathrm{F}A) \rightarrow \mathrm{P}\neg \mathrm{F}A$

(D4) reads "from a thing's being the case it is necessarily follows that it has always been going to be the case". Then we can point out that (D4) is the necessitation of the axiom $A \rightarrow \mathrm{HF}A$ in Lemmon's tense logic $K_t$. (D5) means that of whatever is and always be false, it has already been the case that it will always be false.

The Master Argument reveals that the negation of (M3), i.e. (D3), follows from (D1), (D2), (D4) and (D5). For our exposition, we show the derivation in the Hilbert system below.

| | | |
|---|---|---|
| (1) | $\neg A \wedge \neg \mathrm{F}A$ | (assumption) |
| (2) | $(\neg A \wedge \neg \mathrm{F}A) \rightarrow \mathrm{P}\neg \mathrm{F}A$ | (D5) |
| (3) | $\mathrm{P}\neg \mathrm{F}A$ | (MP), (1), (2) |
| (4) | $\mathrm{P}\neg \mathrm{F}A \rightarrow \mathrm{LP}\neg \mathrm{F}A$ | (D1) |
| (5) | $\mathrm{LP}\neg \mathrm{F}A$ | (MP), (3), (4) |
| (6) | $\mathrm{L}(A \rightarrow \mathrm{HF}A)$ | (D4) |
| (7) | $\mathrm{L}(A \rightarrow \mathrm{HF}A) \rightarrow (\neg \mathrm{MHF}A \rightarrow \neg \mathrm{M}A)$ | (D2) |
| (8) | $\neg \mathrm{MHF}A \rightarrow \neg \mathrm{M}A$ | (MP), (6), (7) |
| (9) | $\mathrm{LP}\neg \mathrm{F}A \rightarrow \neg \mathrm{M}A$ | $(\neg \mathrm{M}\neg = \mathrm{L}, \mathrm{H} = \neg \mathrm{P}\neg)$, (8) |
| (10) | $\neg \mathrm{M}A$ | (MP), (5), (9) |
| (11) | $(\neg A \wedge \neg \mathrm{F}A) \rightarrow \neg \mathrm{M}A$ | (DT), (1), (10) |
| (12) | $\mathrm{M}A \rightarrow (A \vee \mathrm{F}A)$ | (contraposition), (11) |

Here, (MP) denotes *modus ponens* and (DT) the deduction theorem, respectively. The conclusion (12), i.e. the negation of (M3) leads to the logical determinism. What is wrong with the derivation? In the original argument, three statements seem plausible and the derivation is classically licensed. But the only alleged statement appears to be (D1). In addition, (D5) in Prior's extra assumptions also has room to be discussed. In a classical setting, Prior was discouraged with the result, and he moved to some branching time tense logic. In the next section, we will explore another line, i.e. non-classical setting.

## 3. *Three-Valued Modal Tense Logic* $Q_t^m$

The simple lesson from the Master Argument is that if not-A then not-possible-A. Thus, all false statements about the future come to be necessarily false. According to Aristotle's interpretation of contingency that what can be possibly be either true or false, the conclusion of the Master Argument implies Aristotle's determinist argument; see Aristotle [3]. To refute the argument, we need to carefully analyze the derivation presented in the previous section.

Our strategy is to dispense with extra assumptions about time structure and use a version of non-classical logic while the basic ideas in standard modal and tense logic can be preserved. Since both (D1) and (D2) are premises of the argument, they should be accepted. (D4) is not questionable, because it is a thesis of $K_t$. As a consequence, the status of (D5) is of special importance here. In fact, (D5) plays an essential role in the derivation.

Is (D5) a valid principle? It does not seem valid if $A$ is a *future contingent*. It would be then possible to use a three-valued logic for the reconstruction of the Master Argument. The work in Prior [5] explored the idea, in which Łukasiewicz's three-valued logic is used; see [4]. If $A$ is a future contingent (i.e. indeterminate), then $\neg A$ is also indeterminate. We note that both $\mathrm{F}A$ and $\neg\mathrm{F}A$ are also indeterminate. This implies that the premise of (D5), i.e. $\neg A \wedge \neg\mathrm{F}A$, is indeterminate. However, the consequent of (D5), i.e. $\mathrm{P}\neg\mathrm{F}A$ is false. From the truth-table of Łukasiewicz' three-valued logic, we can conclude that (D5) is indeterminate. In other words, (D5) is not valid. In fact, Prior [6, p. 88] said:

> "So the implication 'If $x$ neither nor ever will be $\phi$-ing, then it has been the case that $x$ will never be $\phi$-ing', could have a neuter antecedent and a false consequent, and in that case the implication as a whole, by Łukasiewicz's table, would not be true but neuter. We cannot, therefore, lay down this implication as a logical law, and the 'Master Argument' fails."

This is Prior's solution to the Master Argument. Here, the rejection of (D5) is based on the use of Łukasiewicz's three-valued logic, and Prior did not think that it is philosophically defensible. We believe that Prior appeared to think that the logic of tensed propositions is three-valued but that he later changed the view. Prior thought that three-valued logic is of no help to formalize indeterminate propositions. In fact, Prior [7] remarked that the truth-functional techniques seems simply out of place if indeterminate propositions are involved. Prior's complaint lies in Łukasiewicz's interpretation of $A \wedge B$, i.e. $A \wedge B$ is indeterminate iff both $A$ and $B$ are indeterminate. The trouble for

Prior is about the case $B$ is of the form $\neg A$, since he believed that $A \wedge \neg A$ is plain false $A$ is indeterminate. This seems to be the reason that Prior proceeded other roots in tense logic.

However, we think that Prior's exposition related to future contingents is very interesting. Therefore, we here rework his ideas. Indeed Łukasiewicz's three-valued logic is not suited for our purposes, but a similar effect is also available by changing classical semantics allowing *truth-value gap* differently. To make our idea formal, we propose a new modal tense logic $Q_t^m$ with a Kripke semantics based on Kleene's weak three-valued logic. Of course, we assume that this new logic would have been more acceptable to Prior. But, Prior did not explore it. We now know that Kleene's weak three-valued logic (also due to Bochvar) is famous to many-valued logicians. We thus guess that Prior were not aware of Kleene's weak three-valued logic.

We turn to a formal exposition. The language of $Q_t^m$ includes logical symbols: $\neg$ (negation), $\wedge$ (conjunction), $\rightarrow$ (implication), F (future possibility), P (past possibility), $\mathrm{S_F}$ (future statability), $\mathrm{S_P}$ (past statability), and M (possibility). A formula is defined as usual. Let $A, B$ be formulas and $p_1, ..., p_n$ be propositional variables. We denote by $FOR$ the set of all formulas.

Temporal and modal formulas can be read as follows:

$\mathrm{F}A$ (it will be the case that $A$)
$\mathrm{P}A$ (it has been the case that $A$)
$\mathrm{S_F}A$ ($A$ is statable at all future time points)
$\mathrm{S_P}A$ ($A$ is statable at all past time points)
$\mathrm{M}A$ ($A$ is possible)

Here, a word is need to explain the notion of *statability*. By statability, we mean that a formula has a definite truth-value, i.e. either true or false. From this, $\mathrm{S_F}A$ expresses that $A$ has a truth-value at all future time points.

We can introduce the logical symbols: G (future necessity), H (past necessity), S (statability) and L (necessity), by definition:

$\mathrm{G}A =_{\mathrm{def}} \neg\mathrm{F}\neg A$ (it will always be the case that $A$)
$\mathrm{H}A =_{\mathrm{def}} \neg\mathrm{P}\neg A$ (it has always been the case that $A$)
$\mathrm{S}A =_{\mathrm{def}} \mathrm{S_P}A \wedge A \wedge \mathrm{S_F}A$ ($A$ is statable at all time points)
$\mathrm{L}A =_{\mathrm{def}} \neg\mathrm{M}\neg A$ ($A$ is necessary)

Since we assume *metaphysical modalities*, L and M are modal operators in S5. It is to be noted here that G (H) and F (P) are dually defined, but another definitions can be found in Akama, Nagata and Yamada [2].

The Hilbert system of modal tense logic $Q_t^m$ is the following:

- Axioms for Classical Logic
- Axioms for $\mathrm{S_P}, \mathrm{S_F}$, and S
  - (S1) $\mathrm{S}_* A \to \mathrm{S}_* p$, for any $p$
  - (S2) $(\mathrm{S}_* p_1 \wedge ... \wedge \mathrm{S}_* p_n) \to \mathrm{S}_* A$, where $p_1, ..., p_n$ are all the propositional variables in $A$
  - (S3) $\mathrm{PS_F} A \to \mathrm{S_F} A$
  - (S4) $\mathrm{FS_P} A \to \mathrm{S_P} A$
- Axioms for F and P
  - (T1) $(\mathrm{S_F} p_1 \wedge ... \wedge \mathrm{S_F} p_n \wedge \mathrm{G}(A \to B) \wedge \mathrm{G}A) \to \mathrm{G}B$, where $p_1, .., p_n$ are all the propositional variables in $B$ that are not in $A$
  - (T2) $(\mathrm{S_P} p_1 \wedge ... \wedge \mathrm{S_P} p_n \wedge \mathrm{H}(A \to B) \wedge \mathrm{H}A) \to \mathrm{H}B$, where $p_1, .., p_n$ are all the propositional variables in $B$ that are not in $A$
  - (T3) $A \to \mathrm{GP}A$
  - (T4) $A \to \mathrm{HF}A$
- Axiom for Necessity of the Past
  - (NP) $\mathrm{P}A \to \mathrm{LP}A$
- Rules of Inference
  - (MP) $\vdash A, \vdash A \to B \ \Rightarrow \vdash B$
  - (NECF) $\vdash A \ \Rightarrow \vdash \mathrm{G}A$
  - (NECP) $\vdash A \ \Rightarrow \vdash \mathrm{H}A$
  - (NEC) $\vdash A \ \Rightarrow \vdash \mathrm{L}A$

Here, $*$ is either F, P, or empty. We write $\vdash A$ to mean that $A$ is provable. From the axiomatization, the modal tense logic $Q_t^m$ is viewed as $Q$-like system. In fact, $Q_t^m$ is equivalent to the fusion of Prior's $Q$ and Akama, Nagata and Yamada's [2] $Q_t$ with axiom (NP).

Now, we describe a Kripke type semantics for $Q_t^m$ using three-valued valuation. A *Kripke frame* $\mathcal{F}$ for $Q_t^m$ is a tuple $\langle W, < \rangle$, where $W$ is a non-empty set of possible times, and $<$ is an irreflexive binary relation on $W$. Observe that in our formulation a possible world is identified with a possible time point as suggested by White [9]. We dispense with a binary accessibility relation on a set of worlds, since we suppose S5 modality.

A *Kripke model* $\mathcal{M}$ for $Q_t^m$ is a tuple $\langle \mathcal{F}, V, stat \rangle$, where $\mathcal{F}$ is a Kripke frame and $V$ is a three-valued valuation function from $FOR \times W$ to $\{1 \text{ (true)}, \ 0 \text{ (false)}, \ -1 \text{ (undefined)}\}$ satisfying that for any propositional variable $p, t \in W$, $V(p,t) = 1$ or $V(p,t) = 0$ or $V(p,t) = -1$. $stat$ is a *statability relation* on $FOR \times T$. $stat(A,t)$ reads "$A$ is statable at $t$", which is formally interpreted that for every propositional variable $p$ occurring in $A$, $V(p,t) \neq -1$. In other words, $p$ has a truth-value (either 1 or 0). Based on the machinery, formulas are interpreted by three-valued valuation obeying the so-called Kleene's *weak three-valued logic*. $V$ can then be extended for any formulas as follows.

$$
\begin{array}{lll}
V(\neg A,t)=1 & \text{iff} & stat(A,t) \text{ and } V(A,t)=0\\
V(\neg A,t)=0 & \text{iff} & stat(A,t) \text{ and } V(A,t)=1\\
V(\neg A,t)=-1 & & \text{otherwise}\\
V(A\wedge B,t)=1 & \text{iff} & V(A,t)=V(B,t)=1\\
V(A\wedge B,t)=0 & \text{iff} & stat(A,t) \text{ and } stat(B,t)\\
 & & \qquad \text{and } (V(A,t)=0 \text{ or } V(B,t)=0)\\
V(A\wedge B,t)=-1 & & \text{otherwise}\\
V(\mathrm{F}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \exists s(t<s \text{ and } V(A,s)=1)\\
V(\mathrm{F}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \forall s(t<s \Rightarrow V(A,s)=0)\\
V(\mathrm{F}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{P}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \exists s(s<t \text{ and } V(A,s)=1)\\
V(\mathrm{P}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \forall s(s<t \Rightarrow V(A,s)=0)\\
V(\mathrm{P}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{G}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \forall s(t<s \Rightarrow V(A,s)=1)\\
V(\mathrm{G}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \exists s(t<s \text{ and } V(A,s)=0)\\
V(\mathrm{G}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{H}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \forall s(s<t \Rightarrow V(A,s)=1)\\
V(\mathrm{H}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \exists s(s<t \text{ and } V(A,s)=0)\\
V(\mathrm{H}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{S_F}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \forall s(t<s \Rightarrow stat(A,s))\\
V(\mathrm{S_F}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \exists s(t<s \text{ and } not(stat(A,s)))\\
V(\mathrm{S_F}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{S_P}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \forall s(s<t \Rightarrow stat(A,s))\\
V(\mathrm{S_P}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \exists s(s<t \text{ and } not(stat(A,s)))\\
V(\mathrm{S_P}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{L}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \forall s(stat(A,s) \Rightarrow V(A,s)=1)\\
V(\mathrm{L}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \exists s(stat(A,s) \text{ and } V(A,s)=0)\\
V(\mathrm{L}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{M}A,t)=1 & \text{iff} & stat(A,t) \text{ and } \exists s(V(A,s)=1)\\
V(\mathrm{M}A,t)=0 & \text{iff} & stat(A,t) \text{ and } \forall s(V(A,s)=0)\\
V(\mathrm{M}A,t)=-1 & & \text{otherwise}\\
V(\mathrm{S}A,t)=1 & \text{iff} & \forall s(stat(A,s))\\
V(\mathrm{S}A,t)=0 & \text{iff} & \exists s(not(stat(A,s)))\\
V(\mathrm{S}A,t)=-1 & & \text{otherwise}
\end{array}
$$

Here, some remarks are in order. Propositional connectives are interpreted according to Kleene's weak three-valued matrix. Each tense and modal operator is interpreted at statable time point. To ensure the duality of (tensed) necessity and possibility operators, the interpretation of (tensed) necessity operators is novel. However, without future contingents propositions, all formulas behave classically.

We here note that Prior himself developed the modal system called $Q$ involving the notion of statability; see Prior [6]. In addition, Prior also suggested a temporal version of $Q$ in Prior [7], and Akama, Nagata and Yamada [2] formulated his idea by developing a three-valued temporal logic $Q_t$. Akama and Nagata [1] proposed a three-valued Kripke semantics for $Q$ based on Kleene's weak three-valued logic. The proposed semantical model for $Q_t^m$ can be seen as an extension of such works.

The notion of validity is not standard. Namely, we say that a formula $A$ is *valid*, in symbol $\models A$, iff $V(A,t) \neq 0$ for any $t \in W$ in every Kripke model $\mathcal{M}$. This notion is required to validate all classical tautologies in which the valuation of every propositional variable in them is undefined. This implies that all classical tautologies are valid in $Q_t^m$. We can also define validity in frame analogously.

To motivate $Q_t^m$ as a useful system for the Master Argument, we need to prove that $Q_t^m$ is at least sound with the proposed semantics.

*Soundness Theorem*

$$\vdash A \ \Rightarrow \models A$$

(Proof): It suffices to check that all the axioms are valid and the rules of inference preserve validity. We only consider interesting cases.

First, we check the rules of inference.

(MP): Suppose $A$ and $A \rightarrow B$ be valid but $B$ is not valid. Assume first that every propositional variable in $A$ occurs in $B$. Then, both $A$ and $A \rightarrow B$ can be valuated at $t$. From the validity of $A$ and $A \rightarrow B$, they are not false at $t$. Then, $B$ is also not false at $t$.

Next, suppose that there are some propositional variables $p_1, ..., p_n$ in $A$ not occurring in $B$. Here, we can define a $Q_t^m$-model $\mathcal{M}' = \langle T, <, stat, V' \rangle$ such that $V'(p_i, t) = 1$ for every $i$ and $V(p,t) = V'(p,s)$ if $s \neq t$ or $p \notin \{p_1, ..., p_n\}$. It is then possible to show that $V(B,t) \neq 0$ in $\mathcal{M}$ iff $V(B,t) \neq 0$ in $\mathcal{M}'$ for all $s$. Finally, we can claim that if $A$ and $A \rightarrow B$ are not false at $s$ in $\mathcal{M}'$ then $B$ is also not false at $s$.

(NECF): We can treat (NECF), (NECP) and (NEC) similarly. We here prove (NECF). Assume $A$ be valid. It suffices to show that there is a valuation satisfying $V(A,t) \neq 0$, but $V(\mathrm{G}A, t) = 0$, which implies that $stat(A,t)$ and $\exists t(t < s \text{ and } V(A,s) = 0)$. Since $A$ is valid, $V(A,s) \neq 0$ for all $s \in W$. A contradiction.

We next turn to the proofs of axioms. The proofs of (S1)-S(4) are trivial. (T1)-(T4) can be proved as in standard temporal logic $K_t$. We only show that (T1) is valid. Validity of (NP) should be separately proved.

(T1): Suppose that (T1) is not valid. Then, there is a $Q_t^m$ model such that $V(\mathrm{S_F}p_1 \wedge ... \wedge \mathrm{S_F}p_n \wedge \mathrm{G}(A \rightarrow B) \wedge \mathrm{G}A, t) \neq 0$ and $V(\mathrm{G}B, t) = 0$. From the former, we have:

for all $1 \leq i \leq n$, $stat(p_i, t)$ and $\forall s(t < s \;\Rightarrow\; stat(p_i, s))$ and $stat(A \rightarrow B, t)$ and $\forall s(t < s \;\Rightarrow\; (V(A, s) = 1 \;\Rightarrow\; V(B, s) = 1))$ and $stat(A, t)$
and $\forall s(t < s \;\Rightarrow\; V(A, s) = 1)$

Then, we obtain $V(B, s) = 1$. For the case $V(A, t) = V(A \rightarrow B, t) = -1$, we have $V(B, s) = -1$.

From the latter, we have:

$$stat(B, t) \text{ and } \exists s(t < s \text{ and } V(B, s) = 0)$$

This is a contradiction. Therefore, (T1) is valid.

(NP): Suppose that (NP) is not valid. Then, there is a $Q_t^m$ model such that $V(\mathrm{P}A, t) \neq 0$ and $V(\mathrm{LP}A, t) = 0$. From the first conjunct, we have:

$$stat(A, t) \text{ and } \exists s(s < t \text{ and } V(A, s) = 1)$$

The second conjunct is interpreted as:

$$\begin{array}{ll} & stat(\mathrm{P}A, t) \text{ and } \exists u(stat(A, u) \text{ and } V(\mathrm{P}A, u) = 0) \\ \text{iff} & stat(\mathrm{P}A, t) \text{ and } \exists u(stat(A, u) \text{ and } \forall v(v < u \;\Rightarrow\; V(A, v) = 0)) \end{array}$$

Here, set $u = t$ and $v = s$. Then, $V(A, s) = 1$ and $V(A, v) = V(A, s) = 0$ follow. A contradiction. For the case that $V(A, s) = -1$, it is also contradictory. Therefore, (NP) is valid.

From the soundness theorem, $Q_t^m$ can serve as a logic for the Master Argument. However, a technical problem arises. Using the above concept of validity, we should distinguish between true argument and valid argument. In this sense, (D5) is not true argument. In addition, the conditional future contingent of the form $\mathrm{F}A \rightarrow \mathrm{F}B$ is not true, i.e., indeterminate. The price paid is that $A \vee \neg A$ is also not true. Some people may object to the point. The objection seem plausible in a classical setting, but for future contingents this should not be philosophically defensible. This is because the nature of future contingents lies in the fact they are neither true nor false, implying that $A \vee \neg A$ is not true for any future contingent $A$. Prior [6, p. 86] also pointed out that we need to deny the law of excluded middle and assign to the statement of the form $\mathrm{F}A$ the third truth-value. It is therefore natural to accept that $A \vee \neg A$ is not always true in our logic.

On these grounds, we want to conclude that the Master Argument is not true argument in our modal tense logic. In other words, the Master argument is valid under our weaker notion of validity, but it is not valid in standard classical notion of validity if it involves future contingents. The fact means that classical justification of the argument can be conceptually refuted.

## 4. *Alternative Reconstructions*

There are several alternative approaches to the Master Argument in the literature. *Branching time tense logic* is also due to Prior [7]. To formulate branching time tense logic, we need the notion of *history*, which is a maximal linearly-ordered set of time points. Additionally, we must assume the extra conditions on time structures, e.g. transitivity and connectivity.

Prior proposed two kinds of branching time tense logics, namely *Ockhamist* and *Peircean* ones. The semantics of $\mathrm{F}A$ in branching time tense logic is given with the pair of history and time point, i.e. $(h, t)$ that $A$ is true at some time in the future of $t$ as determined by the history $h$. Additionally, the possibility operator $\mathrm{M}A$ can be evaluated in such a way that it is true at $(h, t)$ iff there is some history $h$ such that $A$ is true at $(h, t)$. For Ockhamist logic, there are three kinds of future operators. The first is $\mathrm{MF}A$ saying that $A$ is true in some possible futures. The second $\mathrm{LF}A$ reads that $A$ is true in all possible futures. The Peircean is in fact equivalent to the second, and Ockhamist logic subsumes Peircean logic. The third is $\mathrm{F}A$ which is the standard future tense operator.

It is obvious that in such branching time tense logics future contingents lack a truth-value. As a result, we can overcome the defect in the Master Argument. However, branching time tense logic involves deep philosophical motivations on time structures and its formalization is more complicated than that of linear time tense logic.

One of the most important works on modal tense logic related to the Master Argument may be found in White [9]. White focuses on philosophical and technical difficulties of (D1), i.e. the necessity of the past by formulating a version of modal tense logic. He argued that the main problem with (D1) is that unrestricted uniform substitution leads the past to transmit to the future without using Prior's extra assumptions. In addition, some formal results on related systems were fully established. However, White did not seem to provide his own solution to the Master Argument.

## 5. *Conclusions*

We discussed the Master Argument based on the new three-valued modal tense logic $Q_t^m$ with a Kripke semantics. Since the Master Argument cannot be justified in the logic, we can avoid logical determinism. Our work is in some sense similar to Prior's work based on Łukasiewicz's three-valued logic. It was then shown that we do not need to consider more complicated (modal) tense logic discussed in the previous section.

One might point out that no philosophical motivation of our semantical rules is provided: the only reason why those rules are proposed seems to be that we can make (D5) undefined. Our response to the objection is that our starting point is to give an interpretation of future contingent proposition as indeterminate. This can be established by incorporating the idea of Kleene's weak three-valued logic into the semantics, and it naturally leads to the consequence that (D5) is undefined.

Prior's reconstruction of the Master Argument assumes discreteness and irreflexivity of time. These assumptions are semantically incorporated into our Kripke frame by imposing the corresponding conditions on the temporal accessibility relation $<$. But the assumption of discreteness, which can be derived using (D5), is the source of a difficulty in the Master Argument. The rejection of (D5) can be thus semantically supported. We also point out that a conception of time as discrete was historically not universal in antiquity; see White [9] for details on the argument on the discreteness assumption. We believe that the only needed assumption on time is irreflexivity. Based on these discussions, the proposed semantics can be *philosophically* grounded.

The completeness of $Q_t^m$ could be established by the variant of standard technique without any difficulty (cf. [1], [2]), but such a result is not philosophically interesting.

It is worth studying other types of non-classical semantics (e.g. supervaluation) or non-classical modal tense logic in relation to the discussion on the Master Argument. We leave the investigation of the issue for further work.

Akama:
1-20-1 Higashi-Yurigaoka
Asao-ku, Kawasaki-shi 215-0012, Japan
E-mail: akama@jcom.home.ne.jp

Murai:
Graduate School of Engineering, Hokkaido University
Kita 13, Nishi 8, Kita-ku, Sapporo 080-8628, Japan
E-mail: murahiko@main.ist.hokudai.ac.jp

Miyamoto:
Department of Risk Engineering
School of Systems and Information Engineering
University of Tsukuba, Ibaraki, 305-8573, Japan
E-mail: miyamoto@risk.tsukuba.ac.jp

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